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Cerenkov excitation of whistler/helicon waves by ionospheric HF heating

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Abstract. A novel scheme for exciting VLF waves and injecting them in the magnetosphere is presented. The scheme is based on Cerenkov excitation of whistler and helicon waves in an altitude range of 80–95 km. Contrary to the traditional Cerenkov excitation which relies on a charge moving at speeds exceeding the local phase velocity of the wave, the present scheme relies on currents moving transversely to its flow direction. The moving current source is created by changing the spatial location of the energy deposition of an ionospheric heater in phase with the wave motion. Representative estimates of the VLF power injected in the magnetosphere are presented.

1. The Ionospheric Whistler and Helicon Modes

The ionospheric plasma at altitudes between 80–140 km constitutes an unconventional uncompensated plasma (Papadopoulos et al., 1983; Ko, et al., 1986). In this altitude range the electrons are magnetized because $\nu_e << \Omega_e$, where ν_e is the electron-neutral collision rate and Ω_e the electron cyclotron frequency. As a result, the dominant electron plasma cross-field current is the Hall current, given by

$$\underline{J}_{eH}(\underline{r}) = \underline{\underline{\sigma}}_{eH}(\underline{r}) \cdot \underline{E}(\underline{r}) \tag{1}$$

$$\underline{\underline{\sigma}}_{eH} = \frac{1}{4\pi}\frac{\omega_e^2}{\Omega_e}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \equiv \sigma_H \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{2}$$

where ω_e is the electron plasma frequency. On the other hand, the ions are essentially a fixed neutralizing background, since $\nu_i >> \Omega_i$, where ν_i is the ion-neutral collision frequency and Ω_i the ion cyclotron frequency. As a result, they cannot balance $\underline{J}_{eH}$ even for $\omega << \Omega_i$. $\underline{J}_{eH}$ remains the dominant plasma current. Neglecting the displacement current and assuming infinite parallel conductivity (i.e. $\omega << \omega_e^2/\nu_e$) the response of the E-region plasma to an externally imposed current $\underline{J}_s(\underline{r},t)$ is

$$\nabla^2 \underline{E} - \nabla(\nabla \cdot \underline{E}) - \frac{4\pi}{c^2}\frac{\partial}{\partial t}\underline{\underline{\sigma}}_{eH} \cdot \underline{E} = -\frac{4\pi}{c^2}\frac{\partial \underline{J}_s}{\partial t} \tag{3}$$

Assuming a homogeneous plasma in the direction transverse to the magnetic field $\underline{B} = B_o \hat{e}_z$, Fourier transforming equation (3) in time and the transverse spatial direction, and writing

$$E_{r,\ell} = E_x \mp iE_y, \quad J_{r,\ell} = J_{sx} \mp iJ_{sy} \tag{4}$$

we find

$$\frac{d^2}{dz^2}E_{r,\ell} - \left(\frac{k_\perp^2}{2} \mp \frac{4\pi\omega\sigma_H}{c^2}\right)E_{r,\ell} + \frac{1}{2}(k_x \mp ik_y)^2 E_{\ell,r} = -\frac{4\pi i\omega}{c^2}J_{r,\ell} \tag{5}$$

If $k_\perp^2 << 8\pi\omega\sigma_H/c^2$, equation (5) decouples to

$$\frac{d^2}{dz^2}E_{r,\ell} \pm \frac{\omega_e^2}{c^2}\frac{\omega}{\Omega_e}E_{r,\ell} = -\frac{4\pi i\omega}{c^2}J_{r,\ell} \tag{6}$$

These equations describe the right (r) and left (ℓ) polarized waves propagating along the ambient magnetic field. By convention, the right hand polarized wave corresponds to the rotation of the electrons. Key characteristics of equation (6) can be seen by examining the dispersion relation for waves with spatial dependence e^{ikz}. We find

$$\left(\frac{kc}{\omega}\right)^2 = \pm\frac{\omega_e^2}{\Omega_e\omega} \tag{7}$$

The +(-) sign corresponds to the $r(\ell)$ mode. For $\omega > 0$, the ℓ-mode is evanescent, while the r-mode propagates with slow velocity for $\omega < \Omega_e$. Electron-neutral collisions can be included by replacing Ω_e of the RHS of equation (7) by $[\Omega_e - (\omega + i\nu)]$. Notice that the r-wave remains a well defined propagating mode even for $\omega << \nu_e$ as long as $\nu_e < \Omega_e$. The r mode is then described by

$$\omega = \frac{k^2c^2}{\omega_e^2}\Omega_e\left(1 - i\frac{\nu_e}{\Omega_e}\right)/\left(1 + \frac{k^2c^2}{\omega_e^2}\right) \tag{8}$$

Equation (8) describes the whistler mode in the frequency range of $\Omega_e > \omega > \Omega_i$ which extends continuously to the helicon mode at $\Omega_i > \omega$.

2. Auroral Currents Driven by HF Heating

The auroral electrojet current system is driven by the solar wind flow past the geomagnetic field. The currents flow along the magnetic field and close by cross-field currents at altitudes between 60–110 km. For a given electric field $\underline{E}_o$ across the ambient magnetic field imposed by the solar wind flow, there are two dominant electron currents, the Hall and the Pedersen currents given by

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$$\underline{J}_{eH} = -\frac{1}{4\pi}\frac{\omega_e^2}{\Omega_e^2}\frac{\Omega_e}{1+\nu_e^2/\Omega_e^2}\underline{E}_o \times \frac{\underline{B}_o}{|\underline{B}_o|} \tag{9}$$

$$\underline{J}_{eP} = \frac{1}{4\pi}\frac{\omega_e^2}{\Omega_e^2}\frac{\nu_e}{1+\nu_e^2/\Omega_e^2}\underline{E}_o \,. \tag{10}$$

An HF heating pulse whose energy is absorbed in the E-region increases the local electron temperature T_e over a region which is taken as Gaussian shape with scalelengths L_x, L_y, L_z. We further assume $\underline{E}_o = \hat{e}_x E_o$. Since $\nu_e \propto T_e$, a cross-field current will be induced due to the temperature increase as follows from equations (9) and (10). If the energy deposition occurs at altitudes above 80 km, where $\nu_e << \Omega_e$, the dominant induced current will be the Pedersen current given by [Papadopoulos, et al., 1989]

$$\Delta\underline{J}_{eP}(\underline{r},t) = \frac{1}{4\pi}\frac{\omega_e^2}{\Omega_e^2}\frac{(T_e - T_o)}{T_o}\nu_e(T_o)\,S(\underline{r},t)E_o\hat{e}_x \tag{11}$$

where T_o is the ambient electron temperature and $S(\underline{r},t)$ represents the spatial profile of the heated region. Modulation of ionospheric currents by using an amplitude or frequency modulated HF pulse has successfully produced ELF/VLF waves at the modulation frequency, (Getmantsev et al., 1974; Kotik and Trakhtengerts, 1975; Stubbe and Kopka, 1977; Chang et al., 1981; Ferraro et al., 1982; Stubbe et al, 1982). In this case, the source $S(\underline{r},t)$ is of the form $S(\underline{r},t) = S(\underline{r})e^{i\omega t}$. Namely, it is stationary in space and oscillatory in time. As discussed in Papadopoulos et al. (1983; 1993) and Papadopoulos, et al. [1989] an horizontal magnetic moment $Me^{i\omega t}$ is induced by the plasma response to the periodic turning on and off of the heater. This magnetic moment in its turn generates the observable fields.

In this letter we examine a different concept in exciting ELF/VLF waves. In this concept we consider excitation by utilizing an unmodulated but moving current source of the type $S(\underline{r} - \underline{u}t)$. A source of this type can be formed by moving the transmitter horizontally or vertically, thereby altering the energy deposition location. If the speed $\underline{u}$ exceeds the phase velocity of the local helicon/whistler eigenmode, the mode is excited by a process analogous to the sonic boom or the Cerenkov emission by a moving charge. However, contrary to the conventional Cerenkov emission in which the current is <u>parallel</u> to the direction of motion, in our case the current is <u>transverse</u> to the direction of motion. The possibility of exciting the earth-ionosphere waveguide by an ionospheric current source moving at superluminous speeds was previously mentioned in two conference abstracts (Kotik et al., 1986; Borisov et al., 1986). Contrary to the superluminous excitation, our paper examines excitation by subluminous sources and focuses on exciting local plasma waves rather than waveguide modes.

3. Cerenkov Helicon/Whistler Excitation by Ionospheric Heating

From equations (4), (6) and (11) we find for the r wave

$$\frac{d^2}{dz^2}\hat{E} + \omega\hat{E} = -i\omega a\,S(z,\omega) \tag{12}$$

In equation (12) we have used dimensionless variables by defining

$$\hat{E} = E_r/E_o,\ t = \Omega_e t,\ \omega = \omega/\Omega_e,$$
$$z = z\frac{\omega_e}{c},\ a = \frac{T_e - T_o}{T_o}\frac{\nu_e(T_o)}{\Omega_e} \tag{13}$$

The physics of the proposed emission can be illustrated by considering a two dimensional source term in (x, z)

$$S(\underline{r},t) = exp\left[-\frac{(x-u_x t)^2}{L_x^2}\right]exp\left[-\frac{(z-u_z t)^2}{L_z^2}\right] \tag{14}$$

This source represents heating by the HF beam moving with speed $\underline{u} = \hat{e}_x u_x + \hat{e}_z u_z$, where x and z are the horizontal and the vertical directions, respectively. The motion in x direction can be accomplished by sweeping the HF transmitter horizontally, while in the z direction by chirping the HF frequency. Such a moving source in the lower ionosphere between 70 to 95 km in altitude can stimulate Cerenkov emission of the helicon waves. The emitted waves propagate along the ambient magnetic field upwards into the magnetosphere and downwards into the earth-ionospheric waveguide. Since the Cerenkov emission is a coherent process, the resulting helicon waves have a well defined frequency which depends on the speed and size of the source motion. A schematic representation of such a process in the ionosphere is shown in Figure 1.

A comprehensive analysis of the problem for the general case $u_x = 0$, $u_z = 0$, including the effect of spatial inhomogeneities will be published elsewhere. We restrict our presentation here to the analytically tractable case of $u_x = 0$, $u_z = 0$. This case besides illustrating the concept is of interest in injecting VLF waves upwards into the magnetosphere. In the limit $L_x >> L_z$ the source function $S(\underline{r},t)$ reduces to

$$S(z,t) = exp\left[-\frac{(z/u_z - t)^2}{\tau^2}\right] \tag{15}$$

where $\tau = L_z/u_z$. In dimensionless units the transform in time of (15) is

$$S(z,\omega) = \sqrt{\pi}\,\frac{\tau}{\Omega_e}\,exp\left(\frac{i\omega z}{u}\right)exp\left[-\left(\frac{\omega\tau}{2}\right)^2\right] \tag{16}$$

Notice that the dimensionless value of the speed u is u = $(u_z/c)\,(\omega_e/\Omega_e)$. From equations (12) and (16) we find

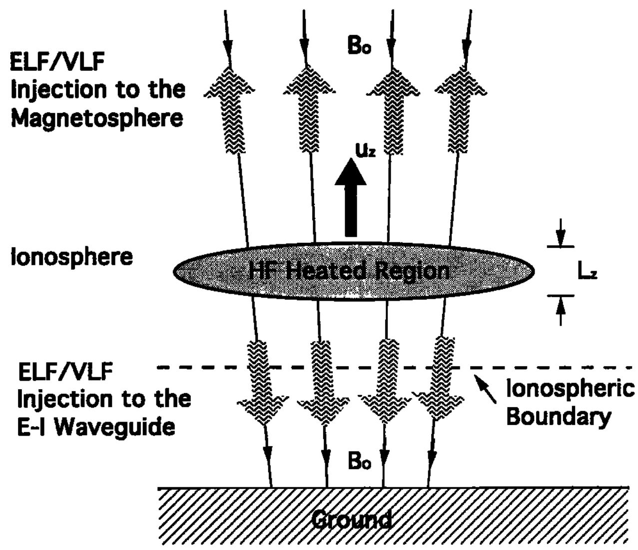

Fig. 1. Cerenkov emission by a moving current source along the magnetic field in the ionosphere.

$$\frac{d^2}{dz^2}\hat{E}+\omega\hat{E} = -\frac{i\omega}{\Omega_e}\sqrt{\pi}(a\tau)exp\left(\frac{i\omega z}{u}\right)\times \\ \times exp\left[-\left(\frac{\omega\tau}{2}\right)^2\right] \tag{17}$$

From the form of the source term it is clear that coherent waves with wavelength ω/u can be excited.

To solve equation (17) we first find the Green's function $\hat{G}(z,z_o)$ as

$$\hat{G}(z,z_o) = -\frac{i}{2\sqrt{\omega}}\ exp[\mp i\sqrt{\omega}(z-z_o)] \tag{18}$$

where $\mp$ corresponds to $z > z_o$ or $z < z_o$, respectively. The electric field produced by the source is then

$$\hat{E}(z,\omega) = -\frac{\pi^{3/2}}{\Omega_e}(a\tau)\sqrt{\omega}\ exp\left[-\left(\frac{\omega\tau}{2}\right)^2\right]\times \\ \times\ \delta\left(\sqrt{\omega}\mp\frac{\omega}{u}\right)e^{\pm i\sqrt{\omega}z}. \tag{19}$$

Notice that the delta function in equation (19) specifies the relationship between the emission frequency, the source velocity u, and the plasma parameters (ω_e, Ω_e). Transforming equation (19) in time and using dimensional units, we find that for $z > z_o$, $u_z > 0$

$$\frac{E_r(z,t)}{E_o} = -\sqrt{\pi}\,a\left(\frac{\omega_o L_z}{u_z}\right)exp\left[-\left(\frac{\omega_o L_z}{2u_z}\right)^2\right]\times \\ \times exp\,\frac{i\omega_o}{u_z}z/exp[-i\omega_o t] \tag{20}$$

where $(\omega_e/\Omega_e)^2(u_z/c)^2$. Equation (20) indicates that the ambient dc electric field E_o has been transformed to a travelling wave with frequency ω_o and wavenumber $k_z = \omega_o/u_z$. The conversion efficiency depends on the modulation level a and the value of $L_z\omega_o/u_z \approx k_zL_z$. It maximizes at $k_zL_z = \omega_oL_z/u_z$. The Poynting flux associated with this wave is

$$P = S_o\pi\left(\frac{T_e-T_o}{T_o}\right)^2\left(\frac{\nu_e}{\Omega_e}\right)^2\left(\frac{c}{u_z}\right)\left(\frac{\omega_oL_z}{u_z}\right)^2\times \\ \times\ exp\left[-\frac{1}{2}\left(\frac{\omega_oL_z}{u_z}\right)^2\right] \tag{21}$$

where $S_o \equiv cE_o^2/8\pi$. Equations (21) describe the transformation of a dc electric field to a broadband travelling helicon/whistler wavepacket with frequency ω_o. Before proceeding we should comment on the efficiency with which the dc electric field E_o can be transformed to an rf field. The factor S_oc/u_z represents a travelling wave with amplitude E_o propagating in a dielectric with refractive index c/u_z. In the absence of amplification it represents the maximum power that can be achieved in a dc to rf conversion. The factor $F = (R_1R_2)^2$ where

$$R_1 = \sqrt{\pi}\left(\frac{T_e-T_o}{T_o}\right)\frac{\nu_e}{\Omega_e} = \sqrt{\pi}\frac{\Delta\nu_e}{\Omega_e} \tag{22a}$$

$$R_2 = \left(\frac{\omega_oL_z}{u_z}\right)exp\left[-\frac{1}{4}\left(\frac{\omega_oL_z}{u_z}\right)^2\right] \tag{22b}$$

represents the conversion efficiency. The factor R_1 depends on the depth of the conductivity or equivalently of the current modulation. Our assumption of predominance of Hall current restricts $\Delta\nu_e/\Omega_e << 1$ and $R_1 << 1$. The factor of R_2 depends on the transit time L_z/u_z of the heater over the modifying spot. It reflects the fact that the typical frequency ω radiated by a moving source of dimension L_z is $\omega \sim u_z/L_z$. The factor R_2 maximizes for $\omega_oL_z/u = \sqrt{2}$ with $R_2 = \sqrt{2}\ e^{-0.5}$. In essence L_z controls the value of the emitted wavelength in the medium. It should be noted that equation (21) apply to the rf power in the generation region. Standard ray tracing and waveguide excitation techniques are needed to describe injection to the magnetosphere and to the earth-ionosphere waveguide.

4. Application to Heating Experiments

We proceed to apply the concept to a situation where ionospheric heating occurs in the 75-90 km range using an HF frequency ω_{HF}. The vertical extent of the source is determined by the characteristic absorption length of the HF wave which is given by $L_z \approx [c/\nu_e(\omega_{HF}/\omega_e)^2]$. For a sweeping speed u_z the factor of R_2 optimizes at a frequency $\omega_o \approx \sqrt{2}\ u_z/L_z$. From these conditions we find

$$\omega_o \approx \sqrt{2}\left(\frac{u_z}{c}\right)\left(\frac{\omega_e}{\omega_{HF}}\right)^2\left(\frac{\nu_e}{\Omega_e}\right)\Omega_e \tag{23}$$

From equation (23) we can see that the wavepacket frequency ω_o can be controlled by the value of u_z/c and the frequency ω_{HF}. If equation (23) is satisfied equation (21) becomes

$$P = \pi S_o\frac{c}{u_z}\left(\frac{T_e-T_o}{T_o}\right)^2\left(\frac{\nu_e}{\Omega_e}\right)^2 \tag{24}$$

For concreteness we assume that the modified region has the following approximate parameters $\omega_e \approx \Omega_e$, $\nu_e/\Omega_e \approx 1/5$, $E_o \approx 10\,mV/m$; we further consider modification at $\omega_{HF} \approx 2\Omega_e$, with $(T_e-T_o)/T_o \approx 1$ and $u_z/c \approx 5\times10^{-2}$. For these conditions $\omega_o \approx 2.4\times10^4$ corresponding to a broadbound VLF wavepacket with frequency 4 kHz. From equation (24) the radiation intensity will be $6\times10^{-7}W/m^2$ which for an horizontal source radius of 10 km, will give radiated power in the 200 W range. This indicates the potential of the Cerenkov process in generating significant powers of VLF waves for magnetospheric injection. We should remark that for the assumed modified region (70–95 km) the required sweeping time is about 1.7 msec which corresponds to about seven wave periods.

5. Summary and Discussion

We presented the analysis of a novel scheme for exciting VLF waves in the lower ionosphere by moving the HF heated region vertically in real time. Physically, this excitation is analogous in nature to the Cerenkov emission by a moving charge in a plasma or to the sonic boom. However, in our case the emitting source moves transversely to the current. The excited VLF waves propagate in the form of an electron right-hand circularly polarized mode (r-mode) along the ambient magnetic field. A more appropriate name for the excited r-mode in the altitude of 80-140 km is helicon. This is because the collisionality freezes the ion motion ($\nu_i >> \Omega_i$) so that the electron r–mode can propagate down to very low frequencies ($\omega < \Omega_i$) without inducing ion currents. This wave is similar to a helicon wave supported by an electron gas in an semiconductor.

It is worth noting that the concept of generating ELF/VLF waves by a moving source in the ionosphere was explored in the past by Kotik et al. (1986) and Borisov et al. (1991). These works differ from the current analysis in one important aspect; their focus was to utilize the moving source to excite the waveguide modes in the earth-ionosphere waveguide. To attain significant excitation, they considered a current source with horizontal motion at superluminus speeds. In this work the emphasis is on the excitation of the local plasma eigenmodes such as the helicon and the whistler waves. As a result subluminal speeds are sufficient to match the phase velocity of the local eigenmodes. Moreover, the source motion is vertical so that the induced radiation can be injected directly into the magnetosphere along the magnetic field lines.

The motion of the current source responsible for the helicon and whistler excitation can be induced by sweeping the HF beam horizontally and/or by chirping the HF pulse in frequency (vertical motion). Both can be readily accomplished by the envisioned heater facility under the HF Active Auroral Research Program (HAARP) (Brandt and Kossey, 1993). We have considered in this letter the vertical motion only in order to illustrate the physical principle of the helicon whistler excitation.

Before closing we should remark on the differences of the proposed technique to the conventional ionosphere generation techniques. In conventional techniques the residence time of the heater on the modified region is long, compared to the transit time of whistler over the absorption length. As a result the induced current closes forming a loop [Chang et al., 1993; Papadopoulos et al., 1993]. The resultant radiation source is a magnetic dipole source. In the Cerenkov case the residence time is short. As a result the equivalent moving source is an horizontal electric dipole which is significantly more efficient. These will be explored in detail in a future publication.

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